N73-2692

DETERMINATION OF THE SPATIAL POSITIONS OF THE LINKS IN A MANIPULATOR MECHANISM

A. V. Sinev, Yu. A. Stepanenko

ABSTRACT. Screw calculus methods are used in an effort to determine the position of a manipulator in space.

One of the main tasks of kinematics that comes up when investigating manipulators is to establish the working space and the work capabilities of the manipulator in different areas within that space, as well as its maneuverability and "service" [1, 2]. /166

Determination of the position of the manipulator in space is basic to these tasks. The complexity of the space mechanism and the great number of degrees of freedom make it necessary to select the most efficient and compact research method. This paper will attempt to use the methods and apparatus of screw calculus for this purpose [3, 5].

Schematic of the Mechanism in the Initial Position and Basic Notation

Let us consider a manipulator mechanism in what we shall call the initial position. The axes of all links are located along one straight line, with the axes of the three hinges parallel to each other and perpendicular to the axes of the links (Figure 1).

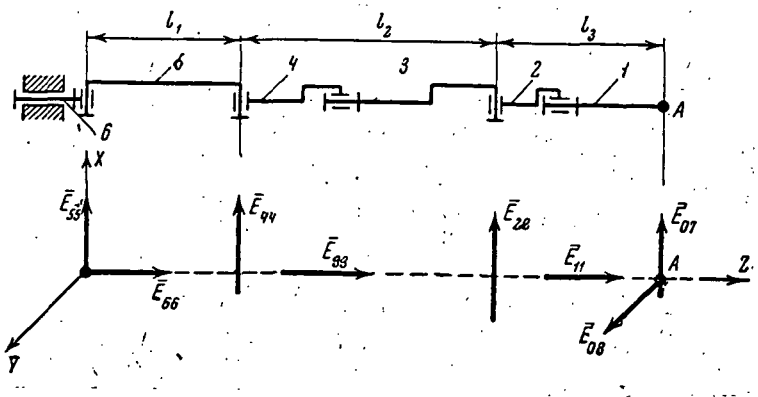

Figure 1. Initial position of mechanism.

Let us introduce the following notation. Let l_1, l_2, and l_3 be the lengths of the links, X, Y, Z the Cartesian coordinates, $\bar{u}_1$, $\bar{u}_2$, and $\bar{u}_1 \times \bar{u}_2$ a set of three unit vectors at pole O and directed along the X, Y, and Z axes. ω is the screw number. /167

Let us introduce the following screws characterizing the initial position of the mechanism: $\bar{E}_{07} = \bar{u}_1 + \omega (l_1 + l_2 + l_3)\bar{u}_2$ in the plane of the tongs, and $\bar{E}_{08} = \bar{u}_2 - \omega (l_1 + l_2 + l_3)\bar{u}_1$ perpendicular to the plane of the tongs. Thus $\bar{E}_{07}$ and $\bar{E}_{08}$ fix the spatial orientation of the tongs at point A.

Screws $\bar{E}_{11}$; $\bar{E}_{22}$; $\bar{E}_{33}$; $\bar{E}_{44}$; $\bar{E}_{55}$ and $\bar{E}_{66}$ characterize the initial position in space of the axes of the six hinges in the mechanism (Figure 1): $\bar{E}_{11} = \bar{u}_1 \times \bar{u}_2$; $\bar{E}_{22} = \bar{u}_1 + \omega (l_1 + l_2)\bar{u}_2$; $\bar{E}_{33} = \bar{u}_1 \times \bar{u}_2$; $\bar{E}_{44} = \bar{u}_1 + \omega l_1 \bar{u}_2$; $\bar{E}_{55} = \bar{u}_1$; $\bar{E}_{66} = \bar{u}_1 \times \bar{u}_2$.

Determination of the Position of the Links in the Mechanism in Space According to the Theory of Final Screw Travel

Let us make a series of successive turns from the initial position (see Figure 1): link 1 (tongs) we turn with respect to the axis $\bar{E}_{11}$ through angle φ_5 (position is counter clockwise); link 2 already turned link 1 through angle φ_4 with respect to axis $\bar{E}_{22}$; links 3, 2, and 1 through angle φ_3 with respect to axis $\bar{E}_{33}$; links 4, 3, 2, and 1 through angle φ_2 with respect to axis $\bar{E}_{44}$; links 5, 4, 3, 2, and 1 through angle φ_1 with respect to axis $\bar{E}_{55}$; and links 6, 5, 4, 3, 2, and 1 through angle φ_0 with respect to axis $\bar{E}_{66}$. Screws in new positions will be designated $\bar{E}_{ik}$, where i is number of turns, and k is the screws index.

The general formula for the final screw turn can be represented in the form [4, 5]

$$\bar{E}_{i+1,k} = \bar{E}_{ik} \cos\bar{\varphi} + (1 - \cos\bar{\varphi})(\bar{E}_{ik} \cdot \bar{E})\bar{E} + \bar{E} \cdot \bar{E}_{ik} \sin\bar{\varphi}, \tag{1}$$

where

$\bar{E}_{ik}$ is the initial position of a single screw;

$\bar{E}_{i+1,k}$ is the final position of a single screw;

E is a single screw in the axis of turn;

$\bar{\varphi}$ is a complex screw turn.

In the case considered we have pure turns because the mechanism contains only rotating couples, so all turns are active.

The position of a single screw can be represented in the form

$$\bar{E}_{ik} = C_{ik}^X \bar{u}_1 + C_{ik}^Y \bar{u}_2 + C_{ik}^Z (\bar{u}_1 \times \bar{u}_2) + \omega [C_{ik}^L \bar{u}_1 + C_{ik}^M \bar{u}_2 + C_{ik}^N (\bar{u}_1 \times \bar{u}_2)], \tag{2}$$

where

i = 0, 1, 2, 3, 4, 5 - the number of turns;

k = 1, 2, 3, 4, 5, 6, 7, 8 - the index of a single screw.

All the new positions of the screws on the horizontal lines in the table were obtained by turning the preceding screws in the kinematic chain through the same angle around a single axis, so we can derive formulas for the sequential calculation of the coefficients C_{ik}.

The screw parameters are calculated by assigning initial values to the coefficients of the reference screw, and then calculating all C_{ik} using the formulas.

Determination of the Position of the Tongs With Respect to a Fixed Coordinate System /168

The position of the tongs, as a solid body, can be assigned six coordinates in space; 3 linear, 3 angular. Let point A of the tongs move to point C with coordaintes (ρ_x, ρ_y, ρ_z). Let us take point C as the origin of the x, y, z system, the axis of which are parallel to the X, Y, Z axes. Let us plot a set of three unit vectors, $\bar{E}_{07}$, $\bar{E}_{08}$, and $\bar{E}_{09}$ from point A to point C, joined to the tongs as to a solid body. Three Euler angles (ψ - precession, θ - nutation, φ - pure rotation) characterize the rotation of a solid body about fixed point C from position with axes x, y, z to position with axes x', y', z' connected to the body. Single screws $\bar{E}_{07}$, $\bar{E}_{08}$ and $\bar{E}_{11}$ will now be directed along axes x', y', z'. Let us designate them $\bar{E}_{67}$, $\bar{E}_{68}$ and $\bar{E}_{61}$ (Figure 2).

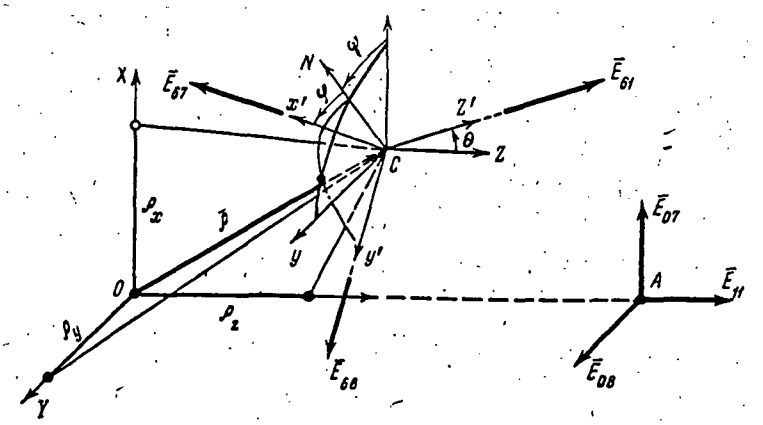

Figure 2. Coordinates of tongs as a solid body in a fixed coordinate system.

The connection between the position of a body with axes x, y, z, and the position with x', y', z' can be expressed by known formulas for nine directional cosines between axes

$$\cos(\widehat{xx'}) = \cos\psi\cos\varphi - \sin\psi\sin\varphi\cos\theta;$$
$$\cos(\widehat{yx'}) = \sin\psi\cos\varphi + \cos\psi\sin\varphi\cos\theta;$$
$$\cos(\widehat{zx'}) = \sin\varphi\sin\theta;$$
$$\cos(\widehat{xy'}) = -\cos\psi\sin\varphi - \sin\psi\cos\varphi\cos\theta;$$
$$\cos(\widehat{yy'}) = -\sin\psi\sin\varphi + \cos\psi\cos\varphi\cos\theta;$$
$$\cos(\widehat{zy'}) = \cos\varphi\sin\theta;$$
$$\cos(\widehat{xz'}) = \sin\psi\sin\theta;$$
$$\cos(\widehat{yz'}) = -\cos\psi\sin\theta;$$
$$\cos(\widehat{zz'}) = \cos\theta.$$

The position of point C relative to the coordinate origin O can be represented by the vector

$$\bar{\rho} = \rho_x\bar{u}_1 + \rho_y\bar{u}_2 + \rho_z(\bar{u}_1\times\bar{u}_2).$$

The active parts of screws E_{67}, E_{68} and E_{61} can be expressed by nine directional cosines ∠169

$$\begin{aligned}
&\mathrm{Re}(\bar{E}_{67}) = \bar{u}_1' = \bar{u}_1\cos(\widehat{xx'}) + \bar{u}_2\cos(\widehat{y}\\
&\mathrm{Re}(\bar{E}_{68}) = \bar{u}_2' = \bar{u}_1\cos(\widehat{xy'}) + \bar{u}_2\cos(\widehat{yy'}),\\
&\mathrm{Re}(\bar{E}_{61}) = \bar{u}_1' = \bar{u}_2' = \bar{u}_1\cos(xz') + \bar{u}_2\cos(\widehat{yz'}) +
\end{aligned} \tag{3}$$

Thus, the position of the screws in space can be represented in the form

$$\bar{E}_{67} = \bar{u}_1' + \omega(\bar{\rho}\times\bar{u}_1');$$
$$\bar{E}_{68} = \bar{u}_2' + \omega(\bar{\rho}\times\bar{u}_2');$$
$$\bar{E}_{61} = (\bar{u}_1'\times\bar{u}_2') + \omega[\bar{\rho}\times$$

from whence, expanding the vector products and using Eq. (3), we obtain, finally

$$\bar{E}_{67} = \bar{u}_1 \cos(\widehat{x x'}) + \bar{u}_2 \cos(\widehat{y x'}) + (\bar{u}_1 \times \bar{u}_2) \cos(\widehat{z x'}) + \\ + \omega \bar{u}_1 [\rho_y \cos(\widehat{z x'}) - \rho_z \cos(\widehat{y x'})] + \omega \bar{u}_2 [\rho_x \cos(\widehat{z x'}) - \rho_z \cos(\widehat{x x'})] + \\ + \omega (\bar{u}_1 \times \bar{u}_2) [\rho_x \cos(\widehat{y x'}) - \rho_y \cos(\widehat{x x'})]. \quad (4)$$

$$\bar{E}_{68} = \bar{u}_1 \cos(\widehat{x y'}) + \bar{u}_2 \cos(\widehat{y y'}) + (\bar{u}_1 \times \bar{u}_2) \cos(\widehat{z y'}) + \\ + \omega \bar{u}_1 [\rho_y \cos(\widehat{z y'}) - \rho_z \cos(\widehat{y y'})] + \omega \bar{u}_2 [\rho_x \cos(\widehat{z y'}) - \rho_z \cos(\widehat{x y'})] + \\ + \omega (\bar{u}_1 \times \bar{u}_2) [\rho_x \cos(\widehat{y y'}) - \rho_y \cos(\widehat{x y'})]. \quad (5)$$

$$\bar{E}_{61} = \bar{u}_1 \cos(\widehat{x z'}) + \bar{u}_2 \cos(\widehat{y z'}) + (\bar{u}_1 \times \bar{u}_2) \cos(\widehat{z z'}) + \\ + \omega \bar{u}_1 [\rho_y \cos(\widehat{z z'}) - \rho_z \cos(\widehat{y z'})] + \omega \bar{u}_2 [\rho_x \cos(\widehat{z z'}) - \rho_z \cos(\widehat{x z'})] + \\ + \omega (\bar{u}_1 \times \bar{u}_2) [\rho_x \cos(\widehat{y z'}) - \rho_y \cos(\widehat{x z'})]. \quad (6)$$

Determination of the Position of the Tongs for Known Angles of Rotation in the Hinges

Let us calculate the values of the coefficients C_{6k}^{Xk}, C_{6k}^{Yk}, C_{6k}^{Zk}, C_{6k}^{Lk}, C_{6k}^{M} and C_{6k}^{N} when k = 1, 7, 8 and let us equate the coefficients obtained to each of six components of screws $\bar{E}_{67}$, $\bar{E}_{68}$, $\bar{E}_{61}$, found using Eqs. (4) - (6) and Eq. (3) when i = 6, k = 1, 7, 8.

We have, for screw $\bar{E}_{61}$, the equality $\cos\theta = C_{61}^{Z}$; $\sin\psi \sin\theta = C_{61}^{X}$, which permit finding angles θ and ψ.

Equating the coefficients of the moment components of screw $\bar{E}_{61}$, we obtain a system of three equations for finding the coordinates of the tongs

the solution of which in matrix form is

/170

$$\begin{bmatrix} \rho_x \\ \rho_y \\ \rho_z \end{bmatrix} = \begin{bmatrix} 0 & \cos(\widehat{x z'}) & -\cos(\widehat{y z'}) \\ \cos(\widehat{z z'}) & 0 & \cos(\widehat{x z'}) \\ \cos(\widehat{y z'}) & -\cos(\widehat{x z'}) & 0 \end{bmatrix} \cdot \begin{bmatrix} C_{61}^{L} \\ C_{61}^{M} \\ C_{61}^{N} \end{bmatrix}$$

Let us find angle φ from the equality of the active components of screw $\bar{E}_{67}$: $\sin\varphi = 1/\cos\theta \cdot C^Z_{67}$. If $\theta = 0$, then $C^X_{67} = \cos(\psi + \varphi)$ and $C^Y_{67} = \sin(\psi + \varphi)$, from whence $(\psi + \varphi)$ can be determined as one angle of rotation.

It is possible to use additional relationships flowing from an identical equality for the active parts of $\bar{E}_{67}$ using Eqs. (4-6) and Eq. 3 in order to do away with ambiguity in determining ψ, θ, φ.

REFERENCES

1. Kobrinksiy, A. Ye., Stepanenko, Yu. A., "Some Problems of Manipulator Theory," V sb. Mekhanika Mashin [In collection: Mechanics of Machines], No. 7-8, "Nauka" Press, 1967.

2. Vinogradov, I. B., Kobrinskiy, A. Ye., Stepanenko, Yu. A., Tyves, L. I., "Features of Manipulator Kinematics and the Volumes Method," This collection.

3. Dimentberg, F. M., Opredeleniye polozheniy prostranstvennykh mekhanizmov [Determining the Positions of Spatial Mechanisms] AN CCCP Press, 1950.

4. Dimentberg, F. M., Vintovoye ischisleniye [Screw Calculus], "Nauka" Press, 1965.

5. Sinev, A. V., Brodotskaya, I. V., Charnaya, I. S., "A Kinematic and Power Analysis of the Spatial Mechanism of a Pneumatic Hydraulic Actuator for Stopcocks in Gas Mains," V sb. Mekhanika Mashin [In collection: Mechanics of Machines], "Nauka" Press, 1969.

Translated for National Aeronautics and Space Administration under contract number NASw-2038 by Translation Consultants, Ltd., 944 S. Wakefield Street, Arlington, Virginia 22204.